Smart Beta: A New Era In Index Investing

Presented by Professor Andrew Clare, Dr Nick Motson and Professor Stephen Thomas

Part 1: Origins

What Is Smart Beta?

ALPHA AND BETA

$$E(R_i) - R_f = \alpha_0 + \beta(E(R_m) - R_f) + \varepsilon_i$$

- Beta
  - An investment portfolio's relationship with market risk
  - More precisely, beta is a measure of the covariance between the returns in excess of the risk free rate

- Alpha
  - The regular addition to return, over and above that element of return that comes from being exposed to the market
- ε_i
  - Luck!!!

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ORIGINS OF SMART BETA TIMELINE

1960s

Building on Markowitz's mean variance analysis Sharpe develops the Capital Asset Pricing Model (CAPM)

1970s

Haugen and Heins find strong negative relationship between return and volatility.

1980s

Banz finds that small cap stocks outperformed large cap stocks
Basu finds low PE stocks generate higher returns relative to high PE stocks

1990s

Jegadeesh and Titman found buying past winners and selling past losers was highly profitable.

2000s

Practitioners begin to launch investment products based on the academic evidence of "anomalies"

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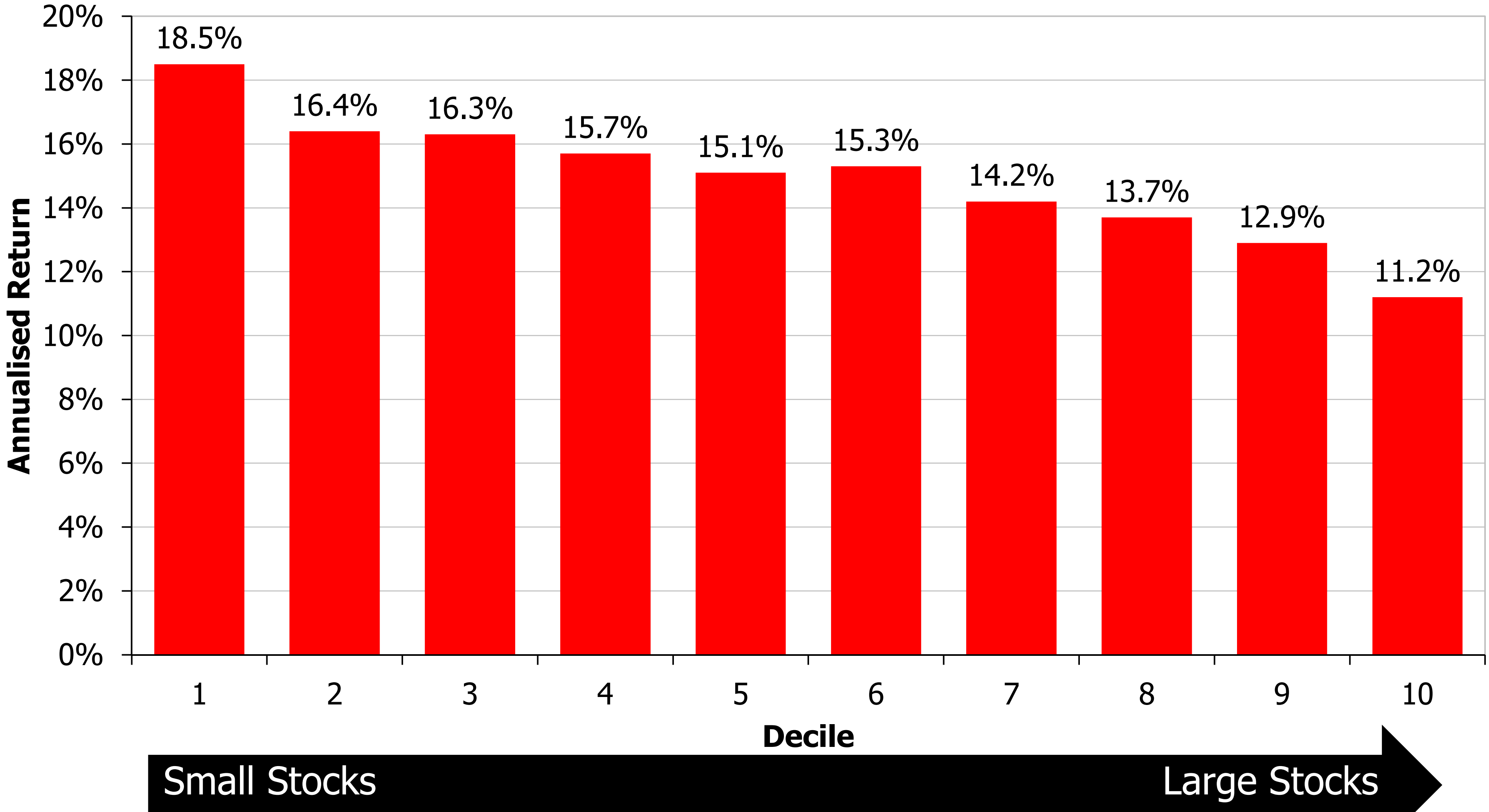
PORTFOLIOS BASED ON SIZE



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Source: Authors' calculations, based upon data available at the Kenneth French website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html

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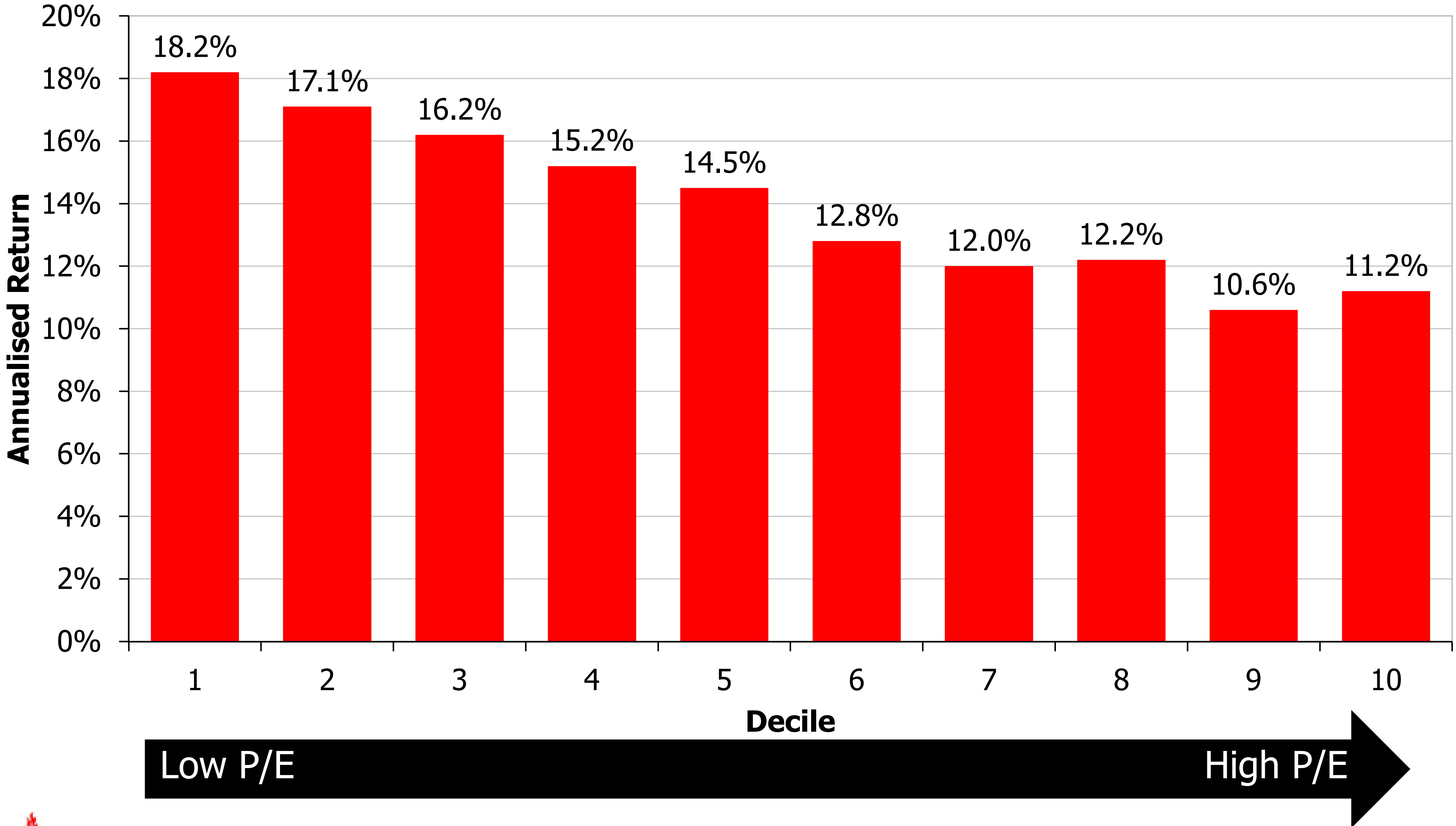
PORTFOLIOS BASED ON PRICE/EARNINGS



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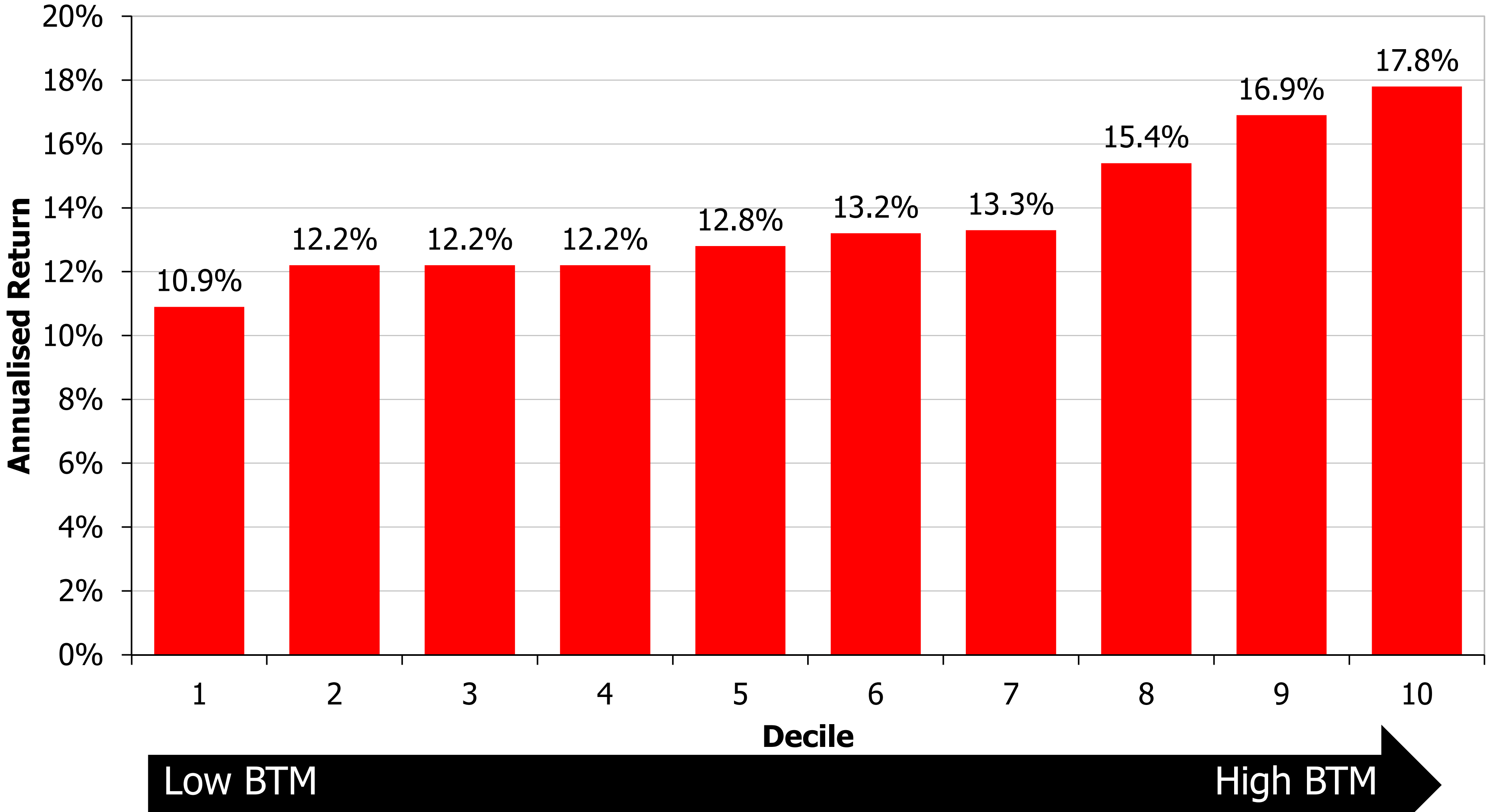
PORTFOLIOS BASED ON BOOK TO MARKET



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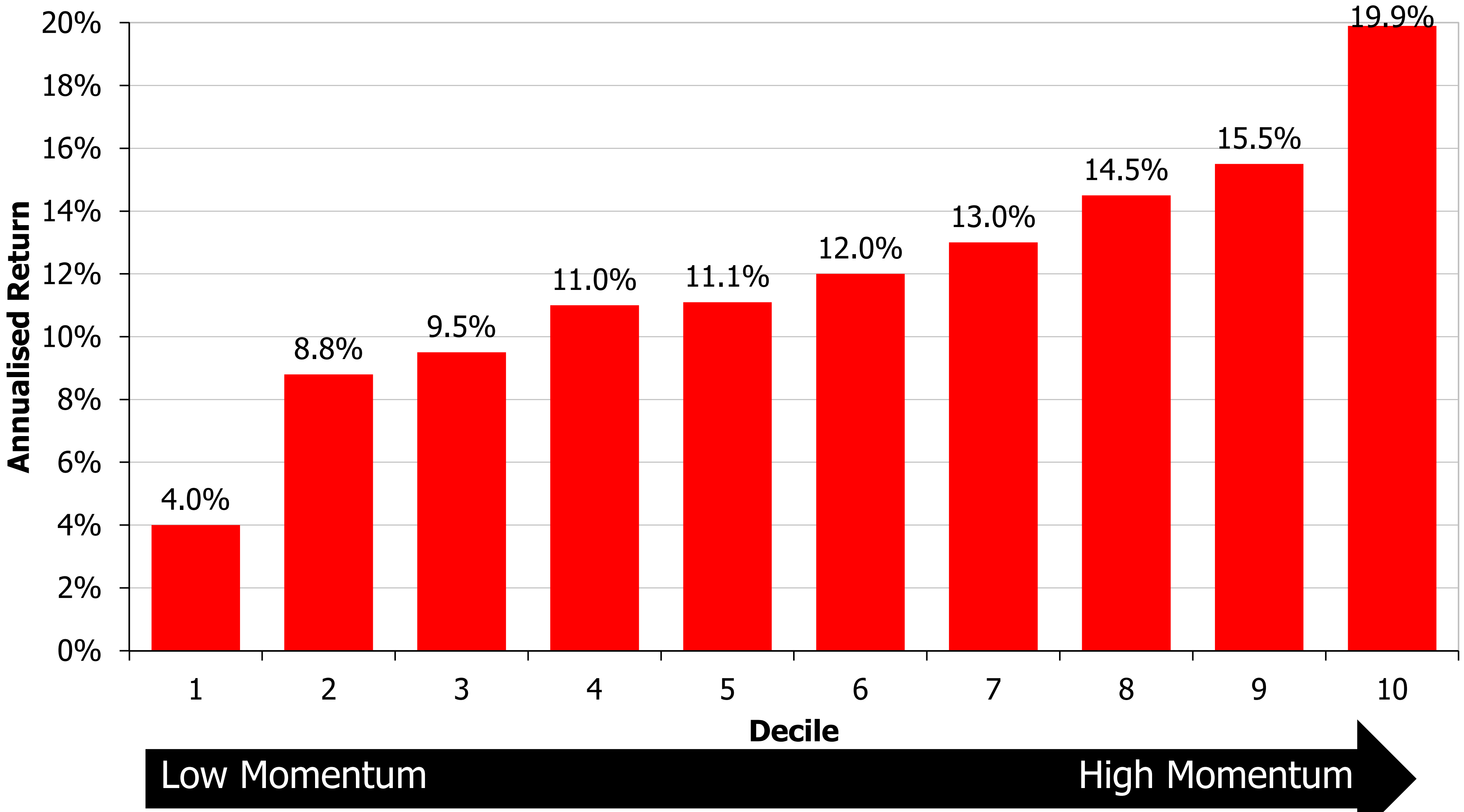
PORTFOLIOS BASED ON MOMENTUM



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Source: Authors' calculations, based upon data available at the Kenneth French website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html

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HOW WERE THESE RESULTS GENERATED?

- The process should look very familiar

(a) At the end of a quarter, consider all the stocks in the London Stock Exchange;

(b) identify the 20% of stocks with the highest dividend yield;

(c) invest in these stocks on either an equally-weighted or a market cap-weighted basis;

(d) hold this portfolio for the following quarter;

(e) at the end of the quarter repeat the process, by once again identifying the 20% of stocks with the highest dividend and investing in these stocks on either an equally-weighted or a market cap-weighted basis;

(f) and then simply repeat this process.

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THIS LED TO OTHER BETAS

- A three factor model

$$E(R_i) - R_f = \alpha_0 + [\beta_1 \times (E(R_m) - R_f)] + [\beta_2 \times (SMB)] + [\beta_3 \times (HML)]$$

- And now a new 5 factor model

$$E(R_i) - R_f = \alpha_0 + [\beta_1 \times (E(R_m) - R_f)] + [\beta_2 \times (SMB)] + [\beta_3 \times (HML)] \\ + [\beta_4 \times (RMW)] + [\beta_5 \times (CMA)]$$

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Part 2: What Lies Beneath?

What Is The Evidence For Smart Beta?

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WHAT WE DID

- US equity data from CRSP
  - Selected 500 largest market cap universe with requirement of 5 years of continuous historical returns each year
  - Sample period December 1968 to December 2014

- Produced indices replicating 8 popular smart beta approaches over a 46 year period
  - Apples to apples comparison

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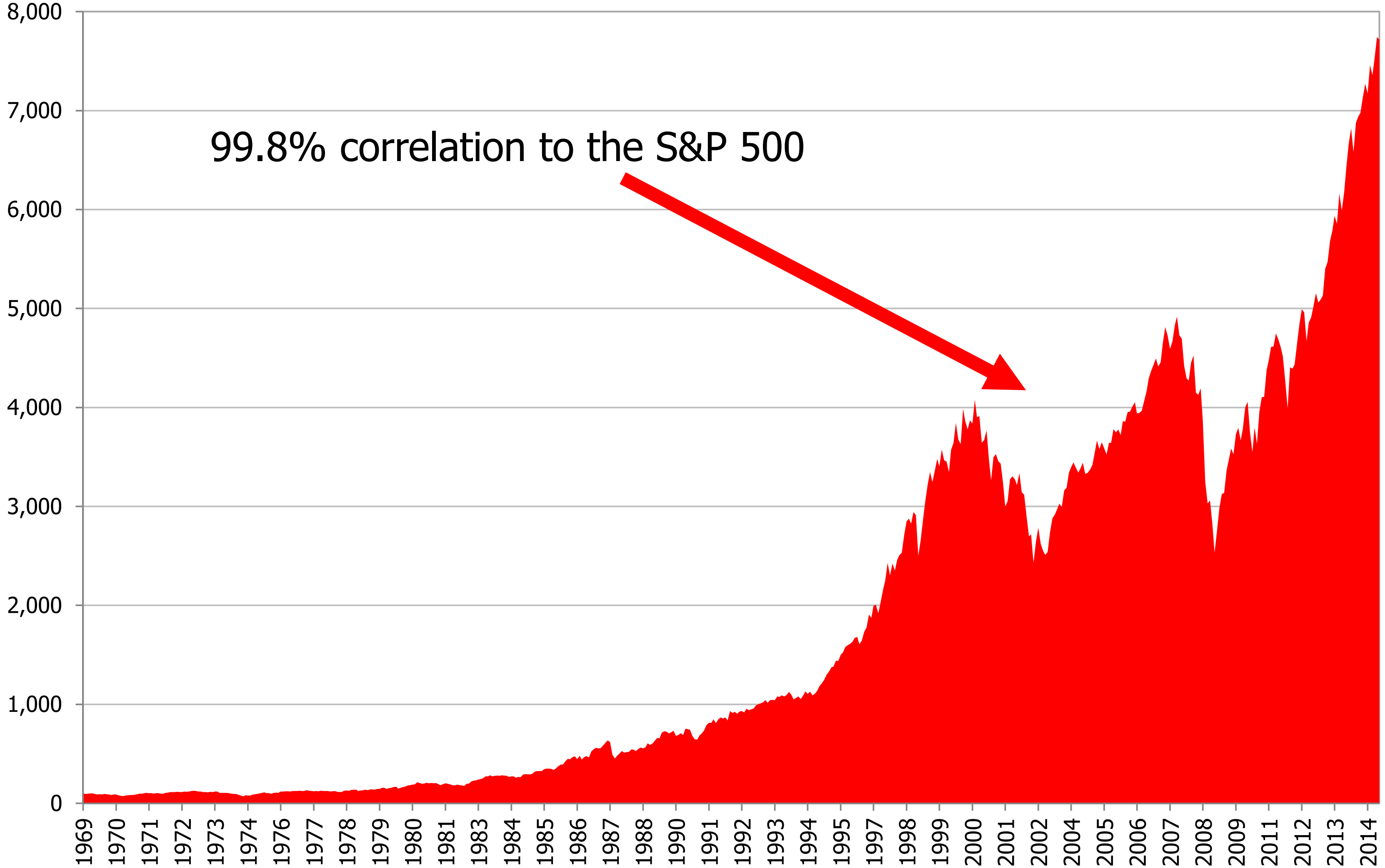
THE BENCHMARK MARKET CAP INDEX



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OUR OWN "SMART" BETA IDEA

- We construct own index using an "innovative" weighting scheme
  - Using the ticker symbol for each stock we calculate the Scrabble™ score for each stock
    - (1 point)-A, E, I, O, U, L, N, S, T, R.
    - (2 points)-D, G.
    - (3 points)-B, C, M, P.
    - (4 points)-F, H, V, W, Y.
    - (5 points)-K.
    - (8 points)- J, X.
    - (10 points)-Q, Z
  - We then sum the scores and divide each stocks score by the total to calculate the weight e.g. XOM has twice the weight of AAPL

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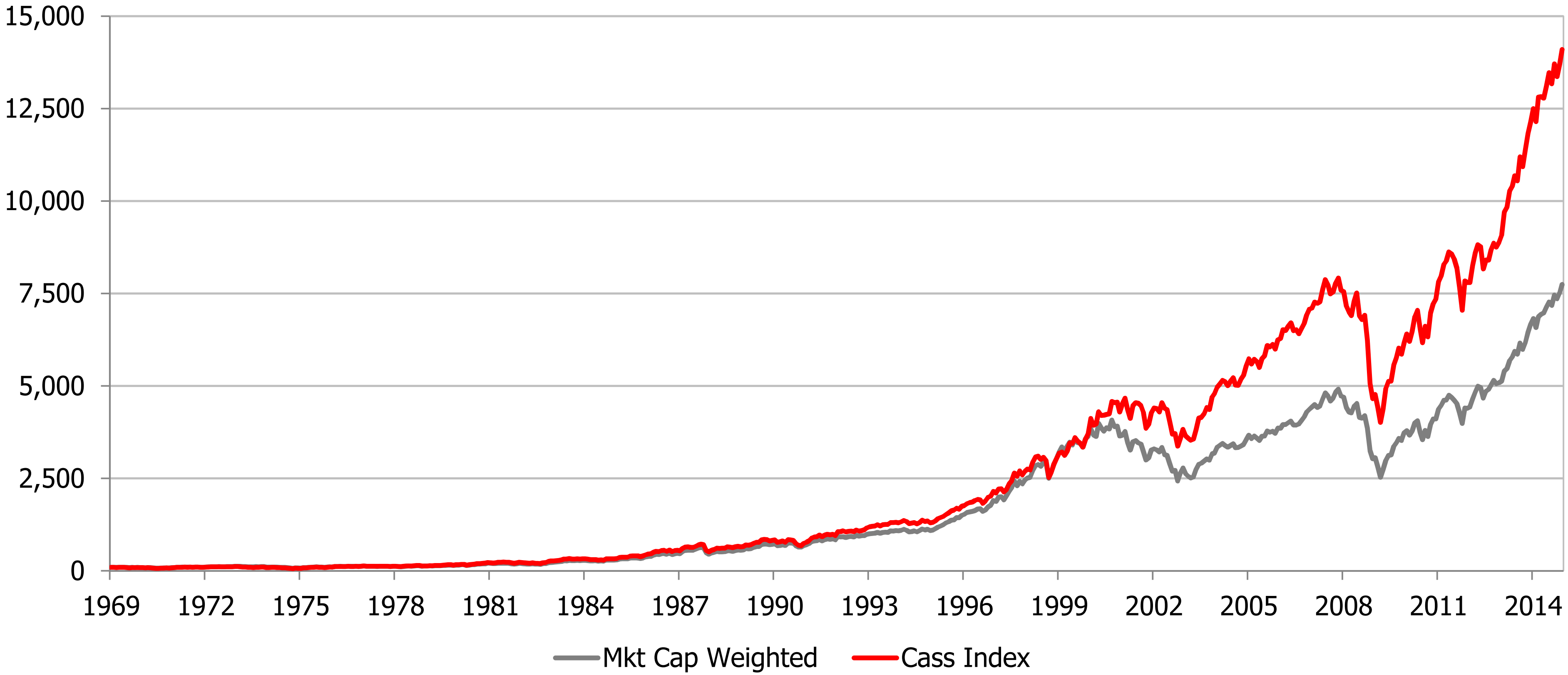
THE CASS INDEX PERFORMANCE



	Terminal Wealth	Mean Return	Standard Deviation	Sharpe Ratio
Market Cap-Weighted	$7,718	10.62%	15.00%	0.38
Cass Scrabble™-Weighted	$14,108	12.15%	16.32%	0.44

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THE IMPLICATIONS OF THIS

- Obviously the Cass Scrabble™ Index is not a real investment proposition
    - Though if you're interested drop me an email

- When evaluating Smart Beta simply looking at the return from a back-test is not enough
    - Need to understand what is driving the returns
    - Is there a reasonable explanation for the historical outperformance?

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THE SET OF ALTERNATIVES

- The eight alternative approached considered:
  - Equally Weighted
  - Diversity Weighted
  - Inverse Volatility
  - Equal Risk Contribution
  - Minimum Variance
  - Maximum Diversification
  - Risk Efficient
  - Fundamentally Weighted

- We followed as closely as possible the index providers methodology but stress we were looking at the spirit as opposed to the law of construction using the academic papers as our guide

**Please see the appendix for the research justifying each alternative as well as the methodology*

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FULL SAMPLE RESULTS 1969-2014 RETURN AND RISK

	Mean Return	Standard Deviation	Sharpe Ratio
Market Cap-Weighted	10.62%	15.00%	0.38
Equally-Weighted	11.93%	16.15%	0.43
Diversity-Weighted	10.98%	15.27%	0.39
Inverse Volatility-Weighted	11.79%	14.13%	0.48
Equal Risk Contribution	11.88%	14.93%	0.46
Minimum Variance Portfolio	10.83%	12.04%	0.49
Maximum Diversification	11.62%	14.16%	0.47
Risk Efficient	12.03%	15.62%	0.45
Fundamentally-Weighted	11.89%	14.81%	0.47

- All 8 of the alternative indices had a higher return
- 5 out of 8 had lower volatility
- All 8 had a higher Sharpe Ratio

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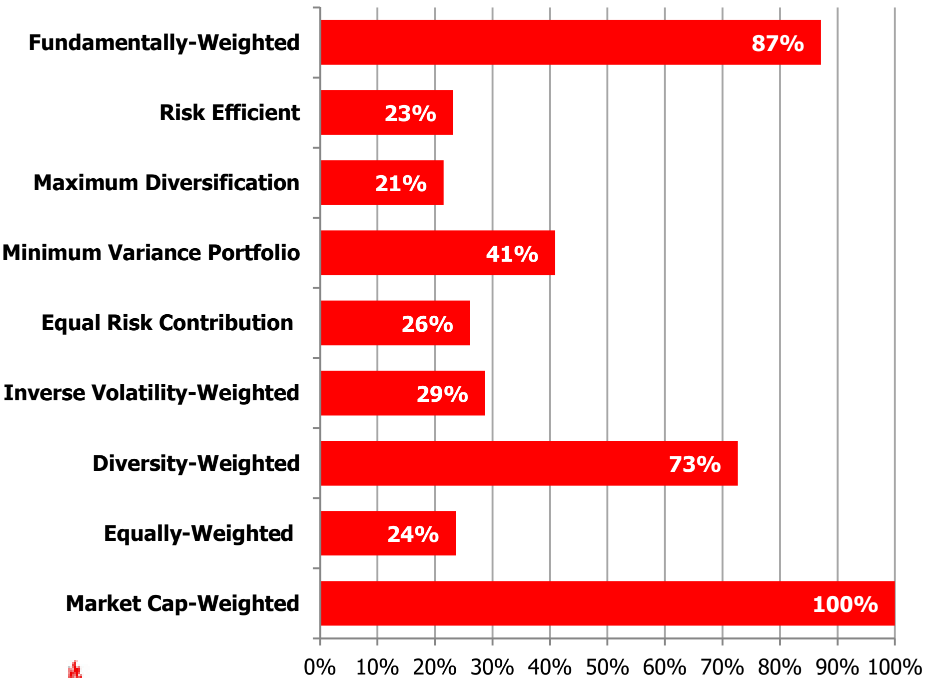
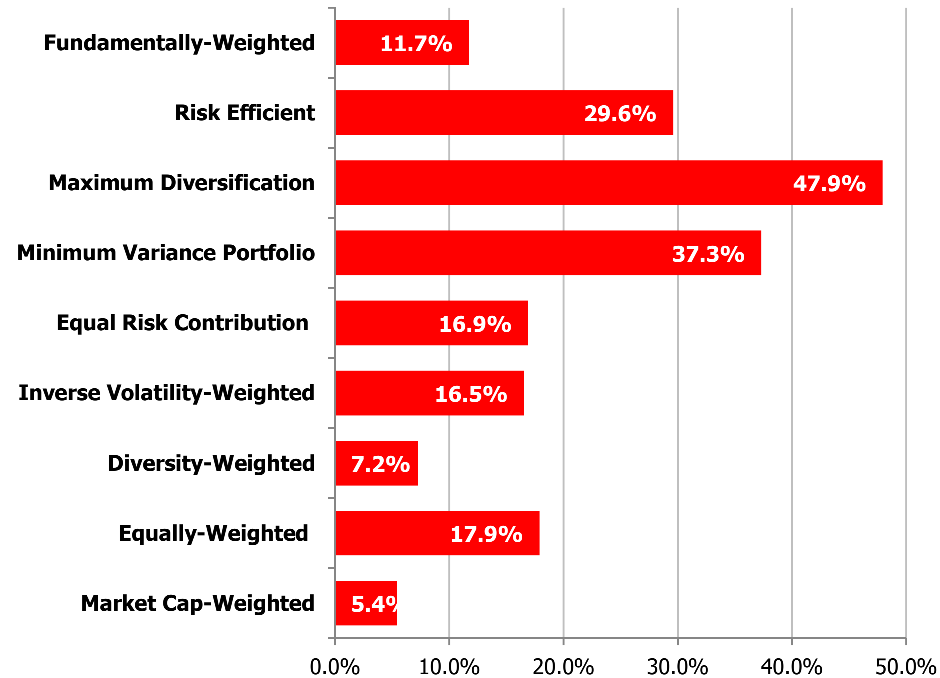
LOOKING UNDER THE SMART BETA HOOD I

- The Smart Beta indices tended to:
  - hold smaller stocks
  - have higher turnover

Weighted Average Market Cap



One Way Turnover



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LOOKING UNDER THE SMART BETA HOOD II

- None of our results incorporate transactions costs, assuming a level of transactions costs could be open to criticism
  - We use the turnover and reverse engineer how high transactions costs would need to be to eliminate the performance difference

	1-Way Turnover	Transaction Cost to Equalize Return	Transaction Cost to Equalize Sharpe Ratio
Market Cap-Weighted	5.4%	-	-
Equally-Weighted	17.9%	5.4%	3.8%
Diversity-Weighted	7.2%	10.3%	7.9%
Inverse Volatility-Weighted	16.5%	5.4%	6.7%
Equal Risk Contribution	16.9%	5.6%	5.8%
Minimum Variance Portfolio	37.3%	0.3%	2.1%
Maximum Diversification	47.9%	1.2%	1.6%
Risk Efficient	29.6%	3.0%	2.6%
Fundamentally-Weighted	11.7%	10.3%	10.7%

  - In our view, in bid-ask spreads would need to have been unbelievably high to have eliminated all of the difference

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EXPLAINING THE OUTPERFORMANCE

	Total	Market	Size	Value	Momentum	Residual
Equally-Weighted	1.32%	0.24%	0.35%	0.60%	-0.26%	0.38%
Diversity-Weighted	0.36%	0.09%	0.09%	0.18%	-0.06%	0.06%
Inverse Volatility-Weighted	1.17%	-0.39%	0.16%	1.16%	-0.09%	0.33%
Equal Risk Contribution	1.26%	-0.11%	0.24%	0.90%	-0.17%	0.40%
Minimum Variance Portfolio	0.21%	-1.94%	-0.01%	1.62%	0.15%	0.40%
Maximum Diversification	1.00%	-0.65%	0.28%	0.61%	0.19%	0.58%
Risk Efficient	1.42%	0.04%	0.34%	0.99%	-0.50%	0.54%
Fundamentally-Weighted	1.27%	-0.01%	0.04%	1.25%	-0.48%	0.48%
Cass Scrabble-Weighted	1.53%	0.25%	0.37%	0.42%	-0.25%	0.75%

- The bulk of the outperformance can be explained by exposure to the factors we already know about

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THE INFINITE MONKEY THEOREM

- The infinite monkey theorem states that *a monkey hitting keys at random on a typewriter keyboard for an infinite amount of time will almost surely type a given text, such as the complete works of Shakespeare.*
- There are an infinite number of possible combinations of portfolio weights for 500 stocks that that would sum to 100%, some of these will outperform the Market Cap approach while others will underperform
- Instead of monkeys devised a robust procedure* to generate 500 random weights that sum to 100% and then relied on some serious computer power to construct 10 million randomly weighted indices

**See Appendix 2 for details of the algorithm*

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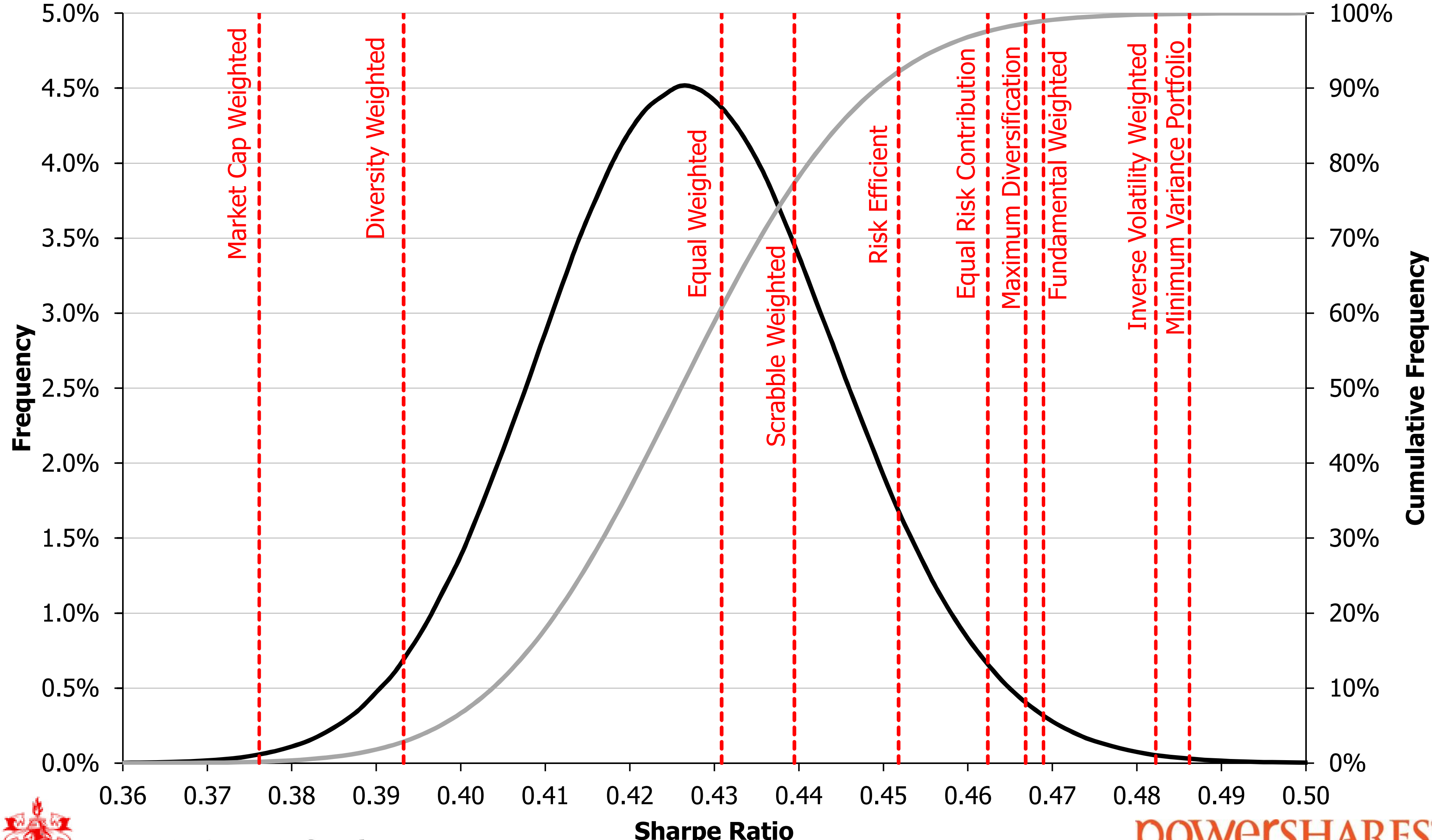
10 MILLION SIMIAN INDICES vs SMART BETA & SCRABBLE INDICES



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CONCLUSIONS

- The back-tested historical risk adjusted returns of "smart beta" indices look good when compared to a market cap weighted index
- The majority of the outperformance can be explained by exposure to value and size factors.
- 99.82% of random (or simian) indices would also have beaten market cap over the same period BUT "smart beta" generally beat over 90% of the monkeys.
- A Scrabble™ weighted index might be a tough sell.

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Part 3: Factors Assembled

Is It Possible To Build Smart Beta Portfolios?

WHAT WE DID

- Nine S&P Smart Beta Indices
  - Sample period December 2001 to September 2015

- What happens if we form ‘passive’ Smart Portfolios?

- Further, can we build ‘Active’ Smart Portfolios?

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THE CANDIDATE INDICES

	Mean return (%pa)	Standard deviation	Sharpe Ratio	Maximum Drawdown
Benchmark				
S&P500	5.90%	14.60%	0.31	50.90%
Factor indices:				
Equal	8.70%	17.30%	0.42	54.90%
Small Cap	9.00%	18.30%	0.41	52.20%
Value	5.60%	15.90%	0.27	56.80%
Momentum	6.30%	14.40%	0.35	44.30%
Low Volatility	8.60%	10.30%	0.70	35.40%
Quality	9.10%	21.10%	0.37	58.60%
Dividend Yield	7.90%	13.80%	0.47	49.30%
Growth	6.10%	14.00%	0.34	45.30%
Low Beta	6.80%	12.40%	0.44	44.70%

Similar to our own indices in paper 2

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FORMING SMART BETA PORTFOLIOS SIMPLE COMBINATIONS

- Equal Weighting
  - 11.11% invested in each of the 9 candidate indices

- Risk Balanced
  - The weight is inversely proportional to the historical volatility of the index
  - Each index weight is 1/σ scaled so the sum is 100%

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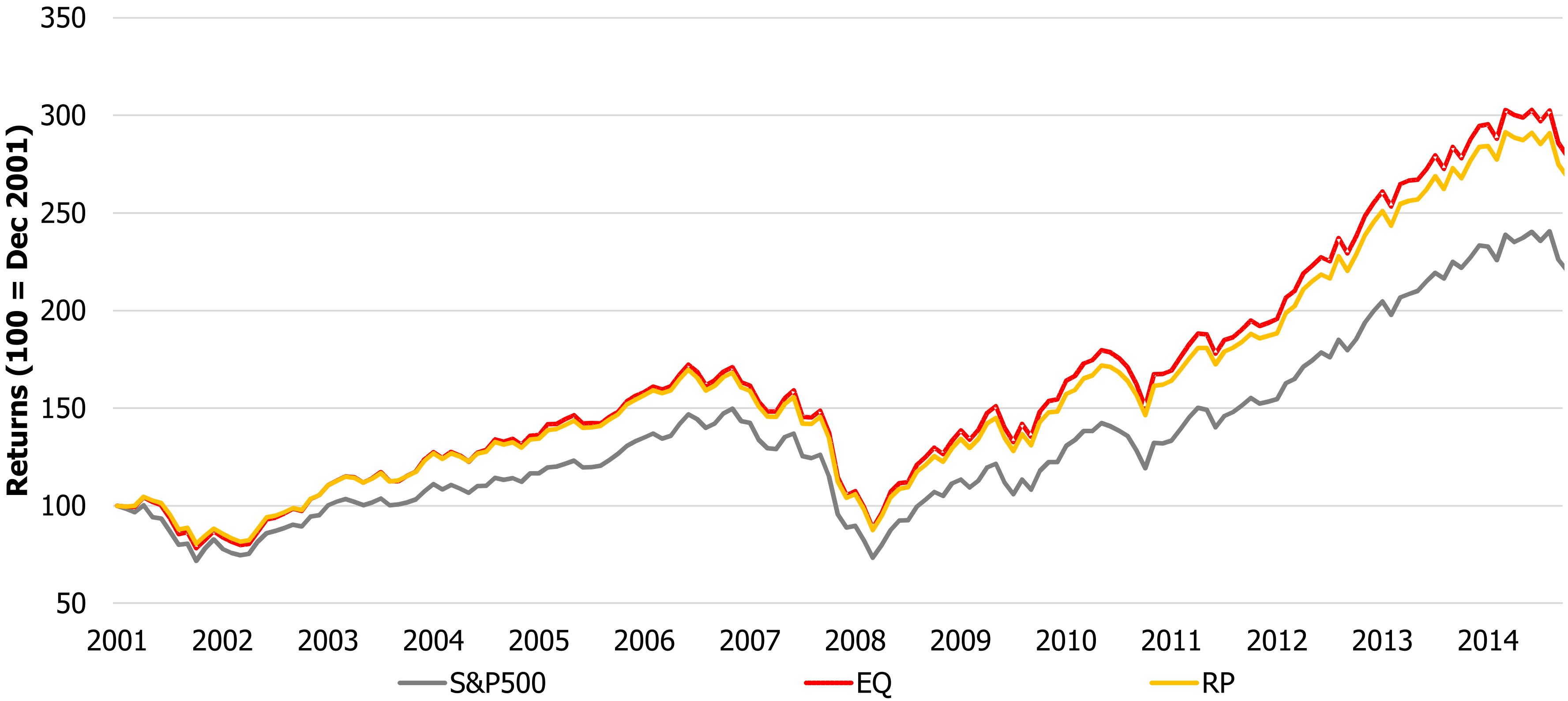
EQUAL WEIGHTED AND RISK BALANCED WEIGHTED PORTFOLIOS



Factor Index Portfolios	S&P500	Equally-Weighted	Risk Balanced
Annualised Returns	5.90%	7.80%	7.50%
Annualised Volatility	14.60%	14.40%	13.70%
Sharpe Ratio	0.31	0.45	0.44
Max. Drawdown	50.90%	48.60%	48.40%

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APPLYING MOMENTUM AND TREND FOLLOWING

- Momentum
  - We construct a simple relative momentum portfolio by taking the best 5 performing strategies over the previous 6 months, and equally weight them (20% in each)
  - This is sometimes called 'cross-section' momentum

- Trend Following
  - Of course the performance of all these strategies may be falling together, so why not only pick those in an upward trend?
  - If the trend is downward, place the said amount in cash or T-Bills
  - This is sometimes called 'absolute momentum' or 'trend following'
  - We determine the trend using an 8 month moving average rule

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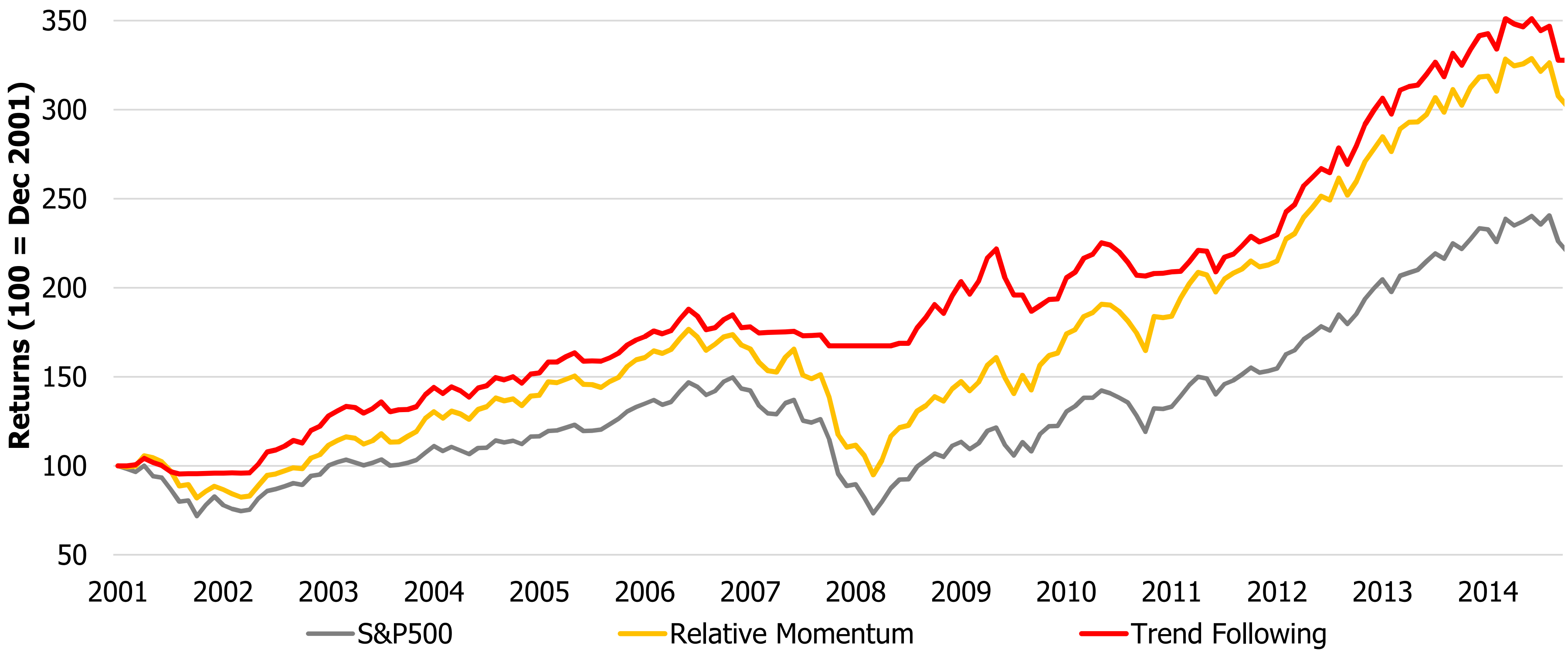
APPLYING MOMENTUM AND TREND FOLLOWING



Factor Index Portfolios	S&P500	Momentum	Trend following
Annualised Returns	5.9%	8.4%	9.1%
Annualised Volatility	14.6%	14.0%	8.9%
Sharpe Ratio	0.31	0.50	0.87
Max. Drawdown	50.9%	46.3%	13.7%

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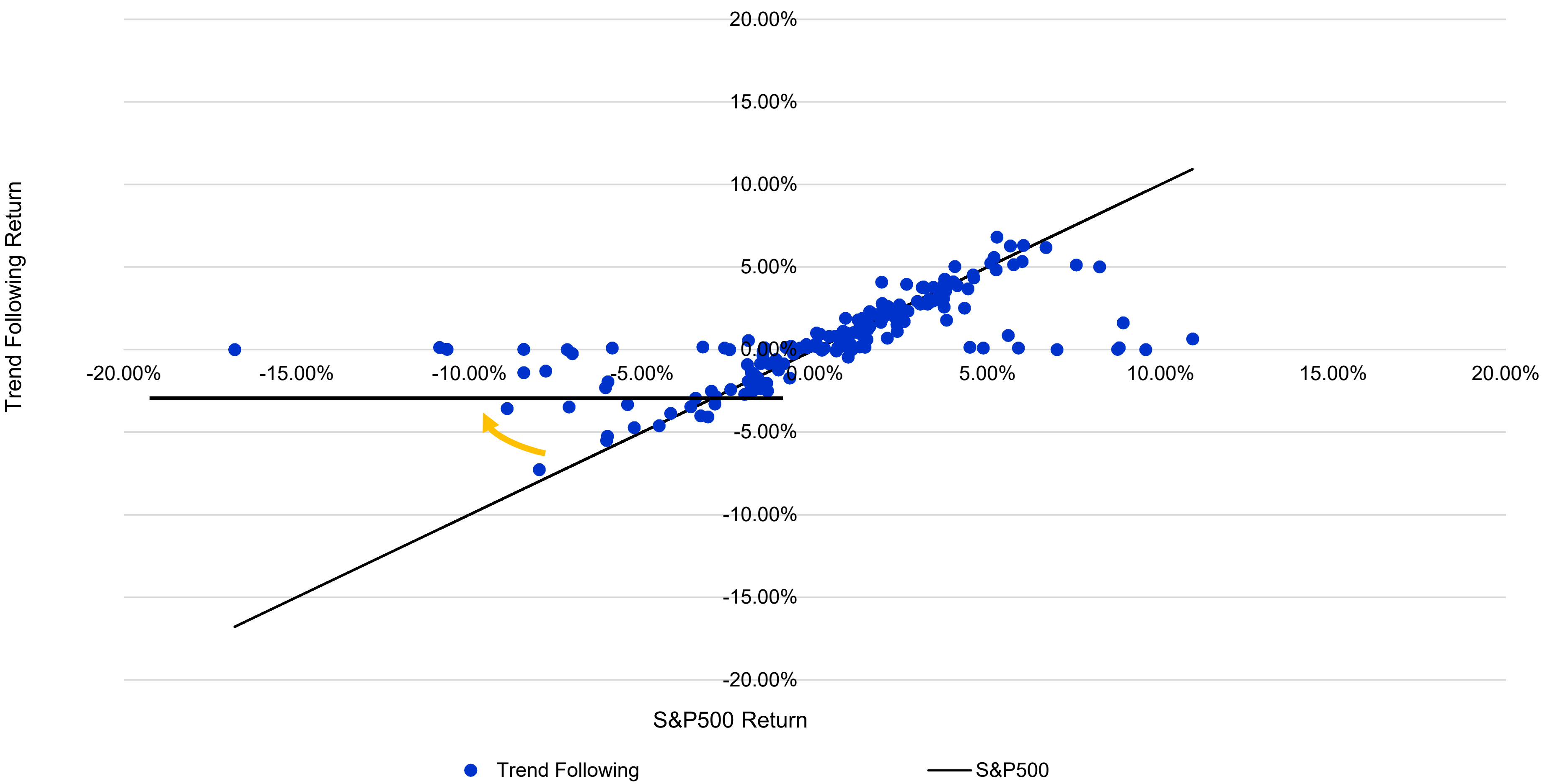
OPTIONALITY IN STRATEGY PAYOFFS



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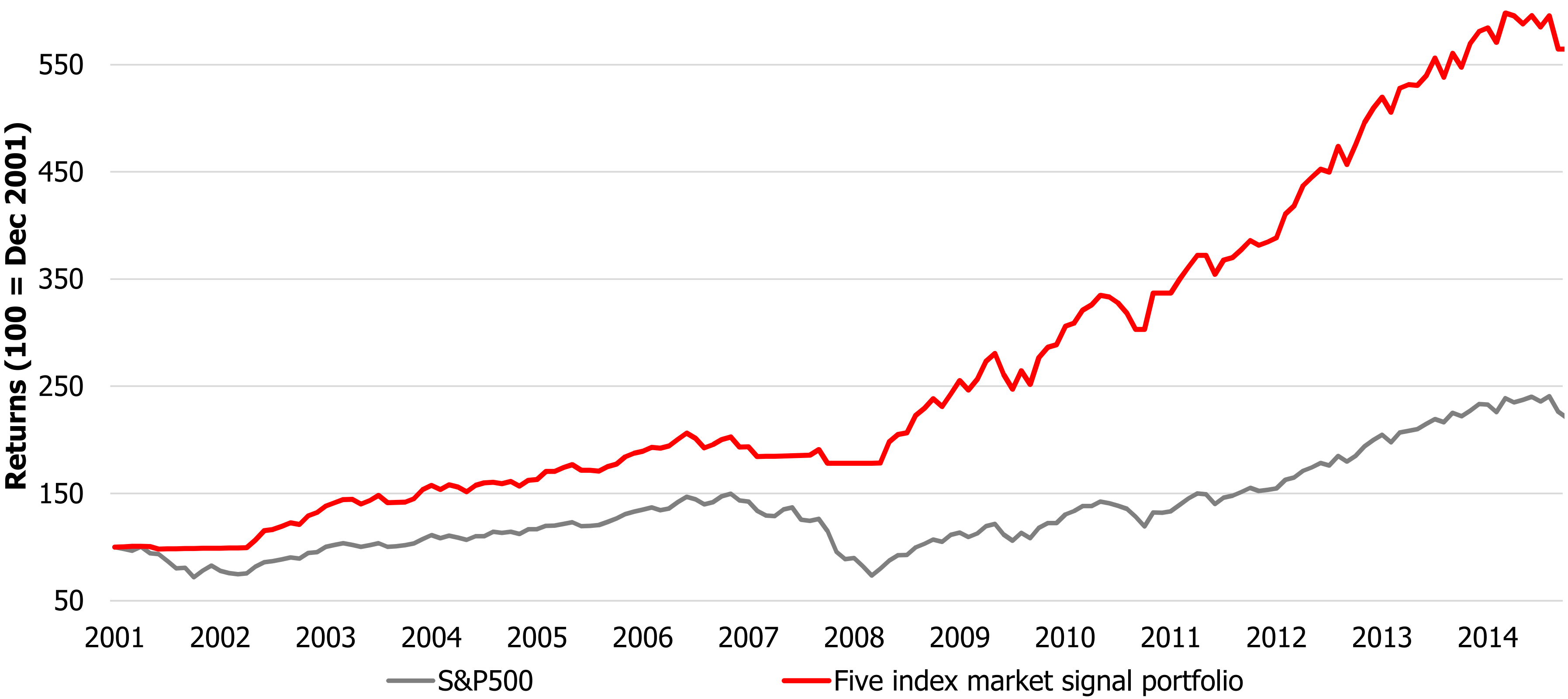
CREATING A MORE DYNAMIC PORTFOLIO

- Is there a role for economics in creating portfolios?
  - There is a growing academic literature relating asset returns to economic variables and bull and bear regimes in stock markets

- We took 5 factor indices (market cap, small cap, value, momentum & low beta) and examined the empirical relation with a small number of forward-looking economic variables, including the VIX and the PMI
  - we then created indicators of bull/bear regimes and use them to switch between T-Bills and the factor indices themselves to form an active equally-weighted portfolio

FIVE SMART BETA INDICES WITH DYNAMIC PORTFOLIO SELECTION



Factor Index Portfolios	S&P500	Equally-weighted	5 factor active portfolio
Annualised Returns	5.9%	8.5%	13.4%
Annualised Volatility	14.6%	14.3%	10.9%
Sharpe Ratio	0.31	0.49	1.10
Max. Drawdown	50.9%	47.5%	13.6%

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Part 4: Monitoring Challenges

How Does Smart Beta Change Investors' Approach To Due Diligence?

AN IDEAL ACTIVE MANAGER?

- According to John Chatfeild-Roberts a good fund manager should:
  - have the necessary skills built into them;
  - be inquisitive, hardworking and ultra-competitive;
  - have the ability to think independently and focus on what's relevant rather than becoming bogged down with irrelevancies;
  - have the humility to admit and rectify mistakes;
  - stick to a proven investment process even when it is not currently working in their favour;
  - be sufficiently experienced, having been exposed to several market cycles; and
  - be in tune with the psychology of the market.

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LESSONS FROM BEHAVIOURAL FINANCE

- Manager behavioural biases:
  - subconsciously create and extrapolate patterns and trends from a series of random events, without investigating the reasons for the apparent trend, known as representativeness;
  - place too much or too little emphasis on the likelihood of an extreme event occurring, based on how easy it is to visualize the event;
  - overestimate one's own investment knowledge, skill and ability, resulting in undiversified portfolios and excessive portfolio turnover to the detriment of investment returns, in other words, the tendency towards overconfidence;
  - leave forecasts unadjusted even in the face of new, contradictory evidence, known as 'adjustment conservatism';
  - place too much emphasis on irrelevant facts and figures, e.g. the price paid for a stock, when considering the stock's future prospects and the price at which to sell, known as 'anchoring'.
  - There are others!

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RULES-BASED INVESTING

- Man vs machine!
  - It is true that rules-based investing can be implemented to eliminate behavioural biases etc
- But
  - Aren't all these rules data-mined and back-tested to death?
  - Is the manager capable of tracking the chosen index/strategy?
  - Is the index provider reputable and reliable? Have they signed up to the IOSCO Principles for Financial Benchmarks?
- But
  - In the end investors are faced with a choice between *rules and discretion*

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Part 5: Recent Work With 7 Global Equity Factors

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INTRODUCTION

- Lessons:

i. Winning strategies change over time

ii. Combinations of strategies perform better

iii. MV analysis suggest concentrated portfolios

iv. ‘Active’ space suggests much more diversified portfolios

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WORLD 6 AND 7 FACTORS

Summary Statistics for 7 Smart Beta Factors

	MSCI World	Equal Weight	Hi Div Yield	Mom	Quality	Risk Weight	Value Weight	Min Vol
Jan 1977 – Nov 2015								
Annualized Return (%)	10.27	11.98	12.10	13.50	11.77	12.61	11.28	-
Annualized Volatility (%)	14.72	15.50	14.20	15.91	14.28	13.54	14.85	-
Sharpe Ratio	0.39	0.48	0.53	0.56	0.50	0.59	0.45	-
Maximum Drawdown (%)	53.65	55.80	58.85	52.48	44.46	50.89	57.52	-
Jan 1977 – Nov 2015 (Relative to MSCI World)								
Annualized Rel. Return (%)	-	1.55	1.66	2.93	1.36	2.13	0.92	-
Tracking Error (%)	-	5.01	6.10	8.16	5.81	5.36	3.33	-
Information Ratio	-	0.31	0.27	0.36	0.23	0.40	0.28	-
Max. Rel. Drawdown (%)	-	36.23	23.21	23.75	36.68	34.53	13.30	-
Jun 1989 – Nov 2015								
Annualized Return (%)	7.10	7.96	9.31	10.15	10.75	9.06	8.24	8.39
Annualized Volatility (%)	15.16	16.04	14.46	15.57	13.89	13.52	15.41	11.39
Sharpe Ratio	0.26	0.30	0.43	0.45	0.55	0.44	0.33	0.46
Maximum Drawdown (%)	53.65	55.80	58.85	52.48	44.46	50.89	57.52	43.01
Jun 1989 – Nov 2015 (Relative to MSCI World)								
Annualized Rel. Return (%)	-	0.80	2.07	2.85	3.41	1.83	1.06	1.21
Tracking Error (%)	-	4.98	6.31	8.12	5.74	5.15	3.40	6.73
Information Ratio	-	0.16	0.33	0.35	0.59	0.36	0.31	0.18
Max. Rel. Drawdown (%)	-	36.23	23.21	20.44	23.41	34.53	13.30	21.93

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CUMULATIVE RETURNS BY FACTOR:6 FACTORS

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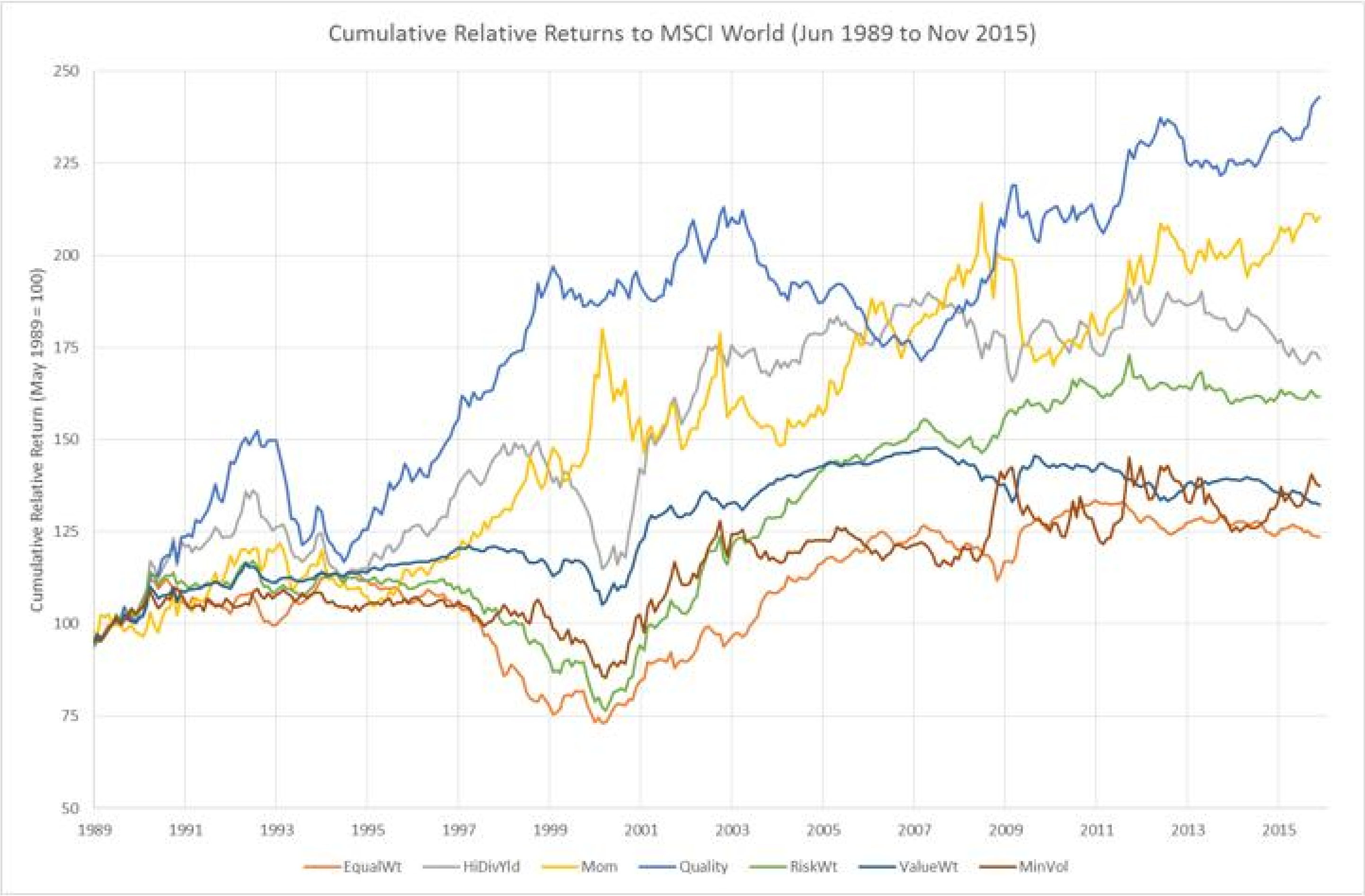
CUMULATIVE RETURNS:7 FACTORS



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COMBINING FACTORS

Portfolios Formed using 7 Smart Beta Factors

	Portfolio Weights (%)								
	Equal Weight	Hi Dividend Yield	Momentum	Quality	Risk Weight	Value Weight	Minimum Volatility		
Jan 1977 – Nov 2015 (Nominal Space)								Annualized Return (%)	Annualized Vol. (%)
Equal Allocation (EA)	16.67	16.67	16.67	16.67	16.67	16.67	-	12.21	13.86
Minimum Variance (MV)	0.00	0.00	0.00	33.47	66.53	0.00	-	12.29	13.22
Maximum Sharpe (MS)	0.00	0.00	33.74	0.00	66.26	0.00	-	12.85	13.56
Risk Parity (RP)	15.49	17.02	16.57	17.31	17.74	15.88	-	12.22	13.83
Jan 1977 – Nov 2015 (Relative Space)								Excess Return (%)	Tracking Error (%)
Equal Allocation (EA)	16.67	16.67	16.67	16.67	16.67	16.67	-	1.66	3.18
Minimum Tracking Error (MT)	8.57	0.00	17.40	10.80	0.00	63.22	-	1.34	2.53
Maximum Information Ratio (MI)	4.05	0.00	25.99	7.79	16.19	45.99	-	1.63	2.84
Active Risk Parity (AP)	17.70	11.29	16.43	17.34	12.88	24.35	-	1.59	2.99
Jun 1989 – Nov 2015 (Nominal Space)								Annualized Return (%)	Annualized Vol. (%)
Equal Allocation (EA)	14.29	14.29	14.29	14.29	14.29	14.29	14.29	9.44	13.43
Minimum Variance (MV)	0.00	0.00	0.00	0.00	0.00	0.00	100.00	8.59	11.29
Maximum Sharpe (MS)	0.00	0.00	0.00	98.25	0.00	0.00	1.75	11.02	13.69
Risk Parity (RP)	12.45	13.83	13.89	14.77	14.53	12.72	17.82	9.45	13.27
Jun 1989 – Nov 2015 (Relative Space)								Excess Return (%)	Tracking Error (%)
Equal Allocation (EA)	14.29	14.29	14.29	14.29	14.29	14.29	14.29	1.95	3.43
Minimum Tracking Error (MT)	12.39	0.00	12.88	15.74	0.00	53.92	5.06	1.71	2.62
Maximum Information Ratio (MI)	15.11	0.00	11.82	39.27	1.31	32.50	0.00	2.24	2.98
Active Risk Parity (AP)	18.00	9.23	13.60	15.99	10.36	22.14	10.68	1.89	3.12

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OUT-OF-SAMPLE:ROLLING ONE-YEAR AHEAD

Out of Sample Portfolio Performance using 7 Smart Beta Factors

Jan 1977 – Nov 2015 (Nominal)	Equal Allocation (EA)	Minimum Variance (MV)	Maximum Sharpe (MS)	Risk Parity (RP)
Annualized Return (%)	12.38	12.84	10.95	12.40
Annualized Volatility (%)	13.92	13.41	13.57	13.90
Sharpe Ratio	0.56	0.61	0.47	0.56
Maximum Drawdown (%)	53.44	52.57	53.94	53.43

Jan 1977 – Nov 2015 (Relative)	Equal Allocation (EA)	Min. Tracking Error (MT)	Max Information Ratio (MI)	Active Risk Parity (AP)
Annualized Relative Return (%)	1.88	1.16	1.08	1.74
Tracking Error (%)	3.21	2.70	2.62	3.15
Information Ratio	0.58	0.43	0.41	0.55
Max. Relative Drawdown (%)	9.04	8.99	6.81	9.84

Jun 1989 – Nov 2015 (Nominal)	Equal Allocation (EA)	Minimum Variance (MV)	Maximum Sharpe (MS)	Risk Parity (RP)
Annualized Return (%)	9.24	8.94	9.61	9.31
Annualized Volatility (%)	13.56	11.56	14.31	13.43
Sharpe Ratio	0.45	0.51	0.45	0.46
Maximum Drawdown (%)	52.04	43.04	54.71	51.58

Jun 1989 – Nov 2015 (Relative)	Equal Allocation (EA)	Min. Tracking Error (MT)	Max Information Ratio (MI)	Active Risk Parity (AP)
Annualized Relative Return (%)	2.00	1.03	1.39	1.72
Tracking Error (%)	3.44	2.78	3.37	3.26
Information Ratio	0.58	0.37	0.41	0.53
Max. Relative Drawdown (%)	9.02	12.60	16.93	10.86

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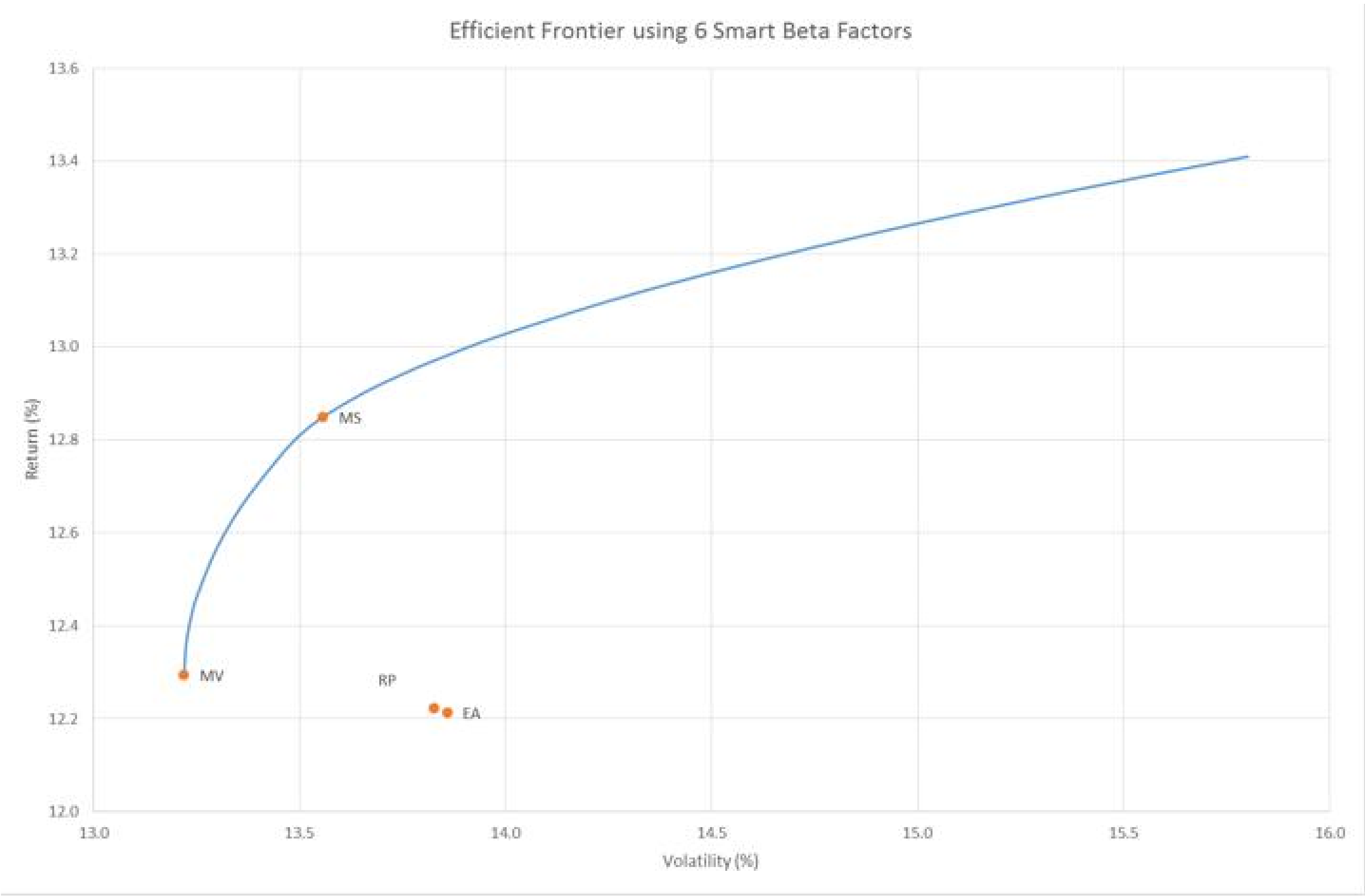
EFFICIENT FRONTIER:6 FACTORS



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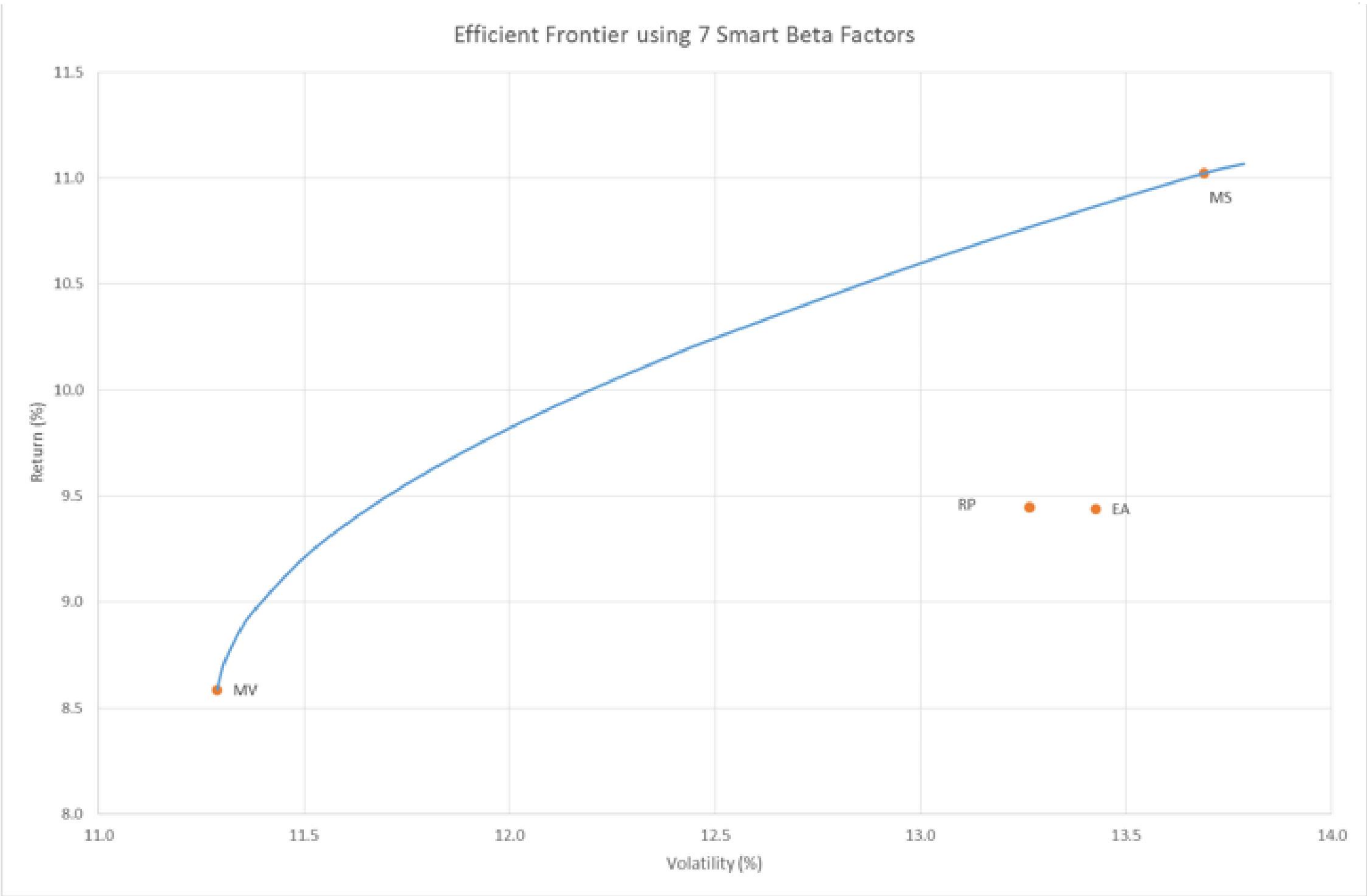
EFFICIENT FRONTIER:7 FACTORS



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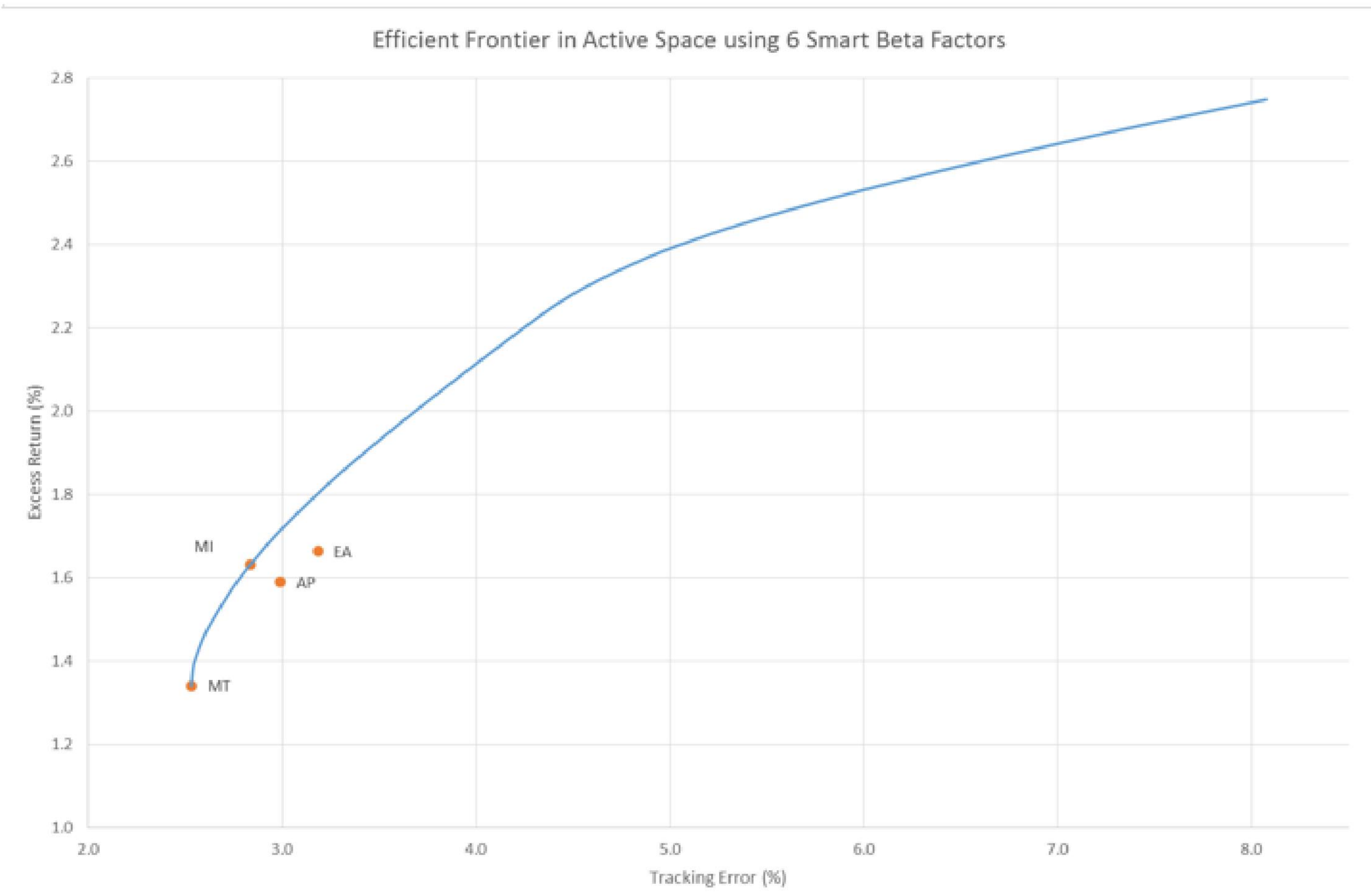
ACTIVE SPACE: EFFICIENT FRONTIER:6 FACTORS



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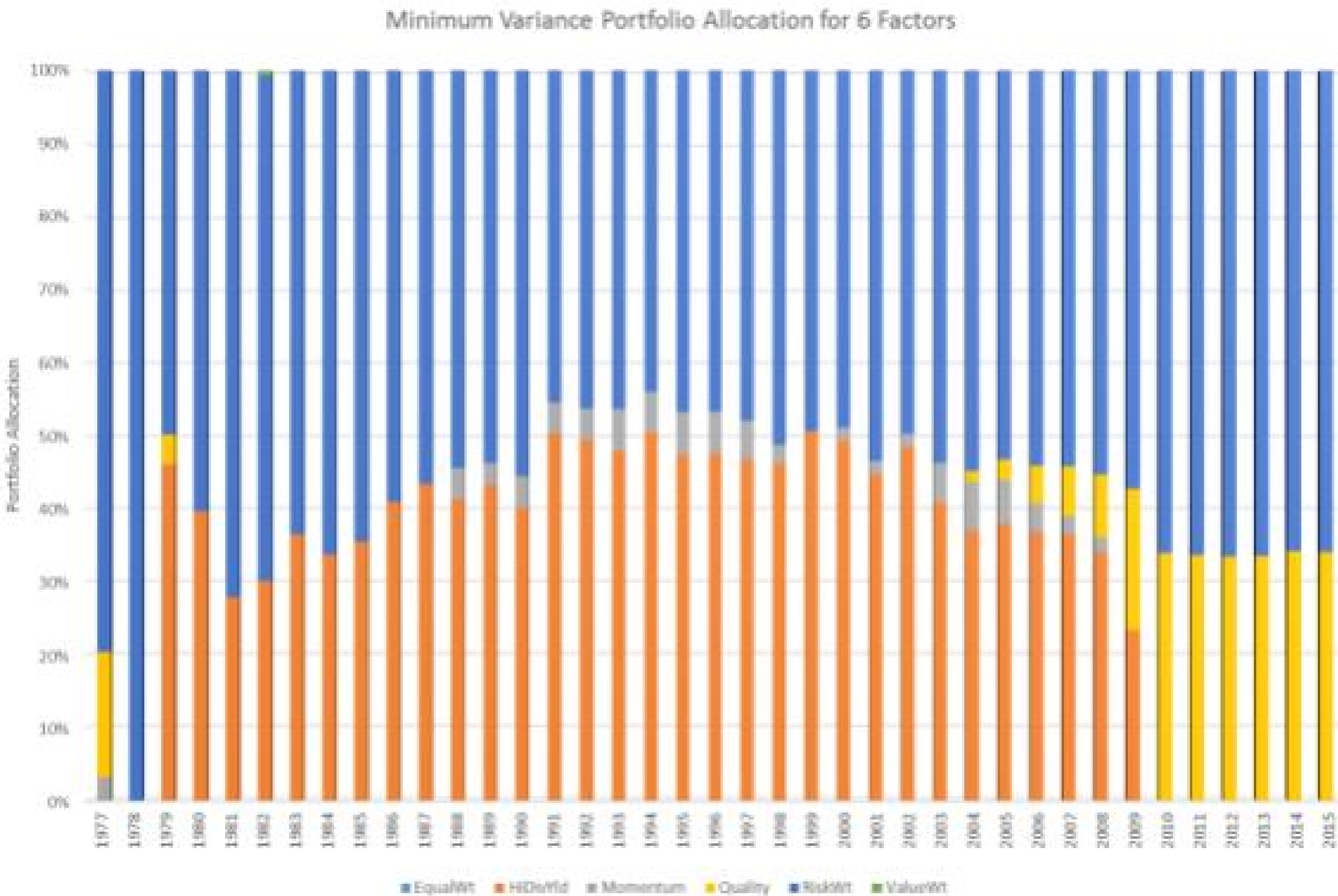
6 FACTORS: WHICH ONES IN A DYNAMIC MIN VARIANCE PORTFOLIO?



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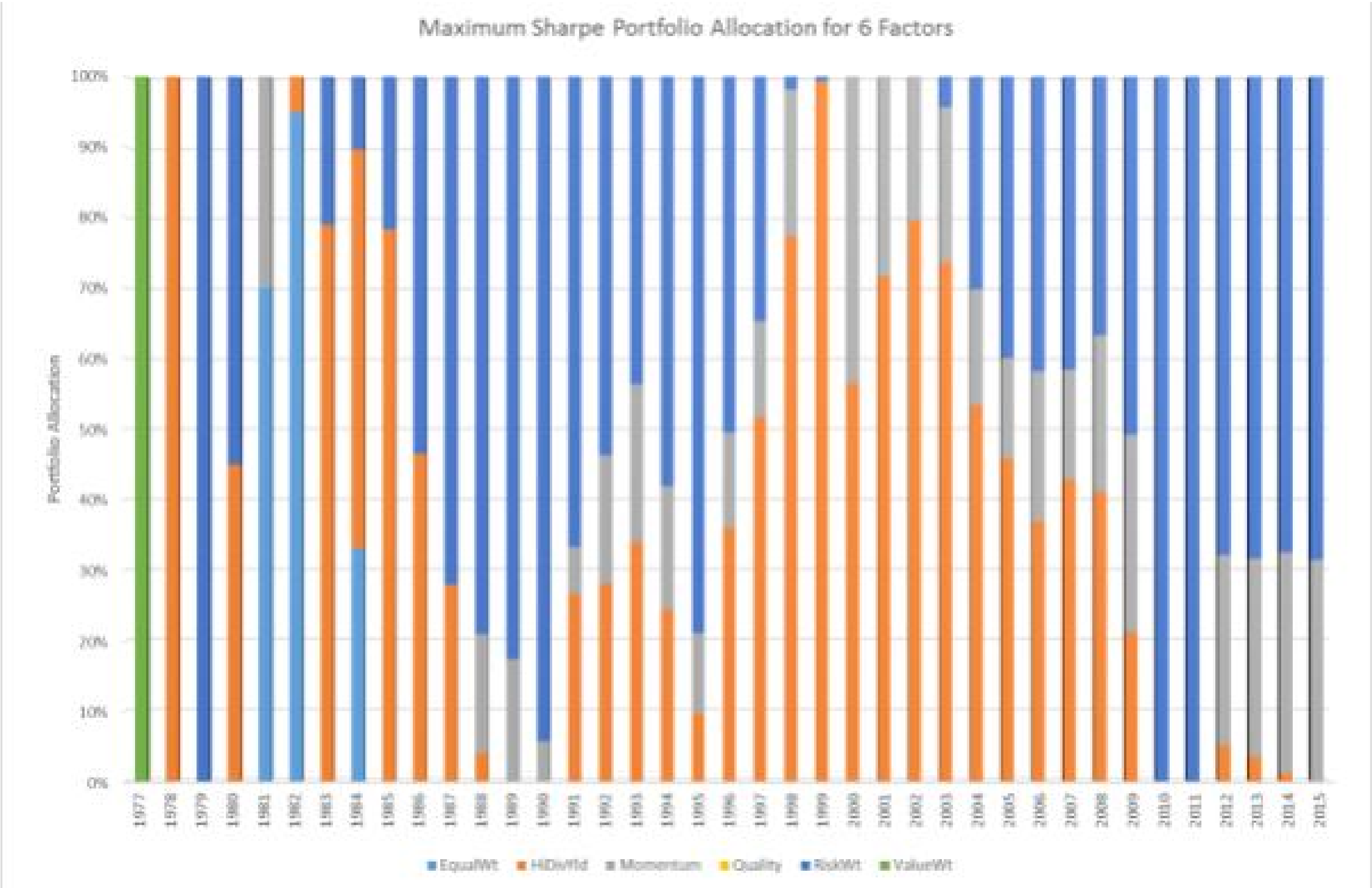
6 FACTORS: WHICH ONES IN A DYNAMIC MAX SHARPE?



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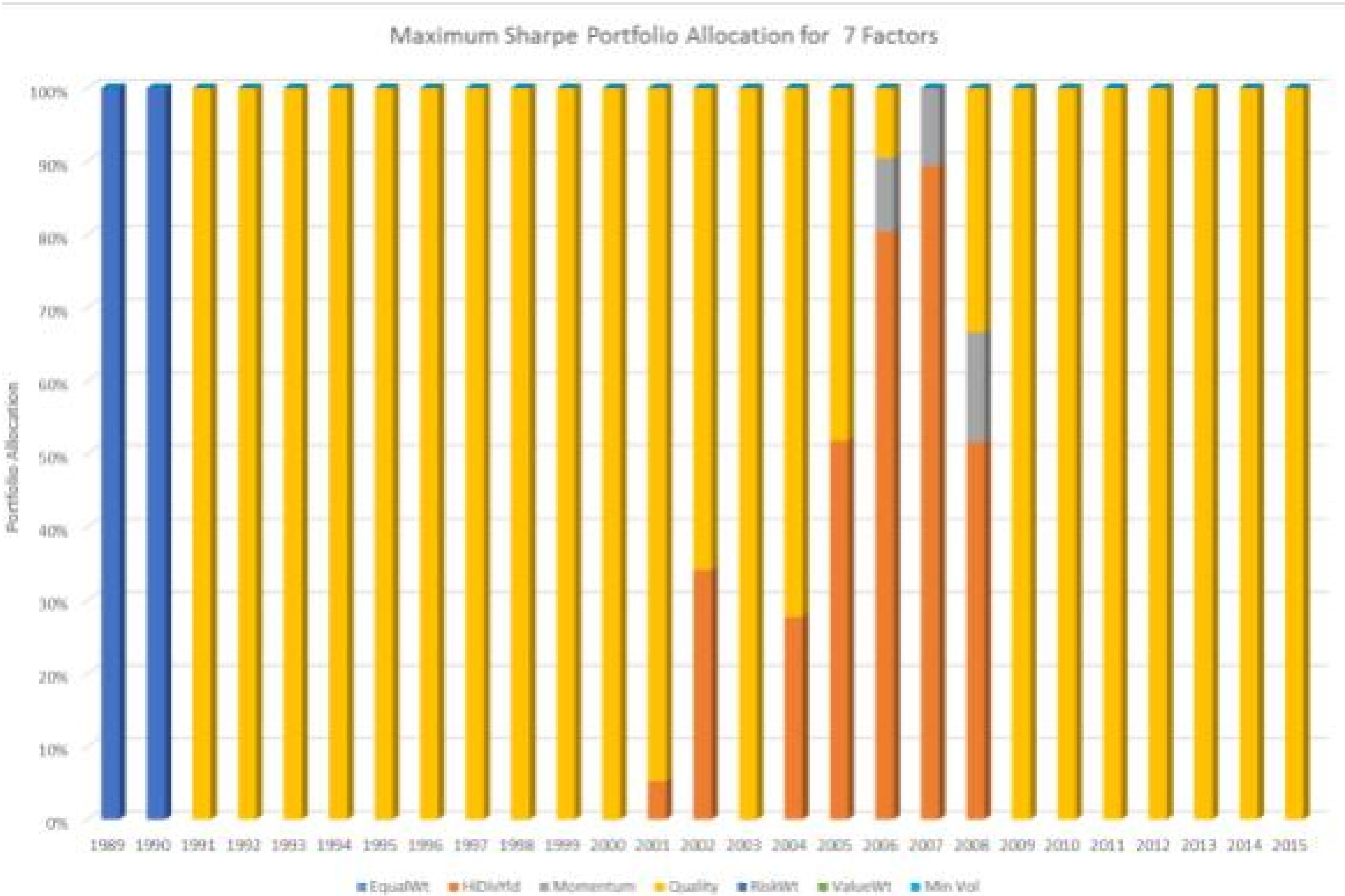
7 FACTORS: MAX SHARPE, WHICH ONES?



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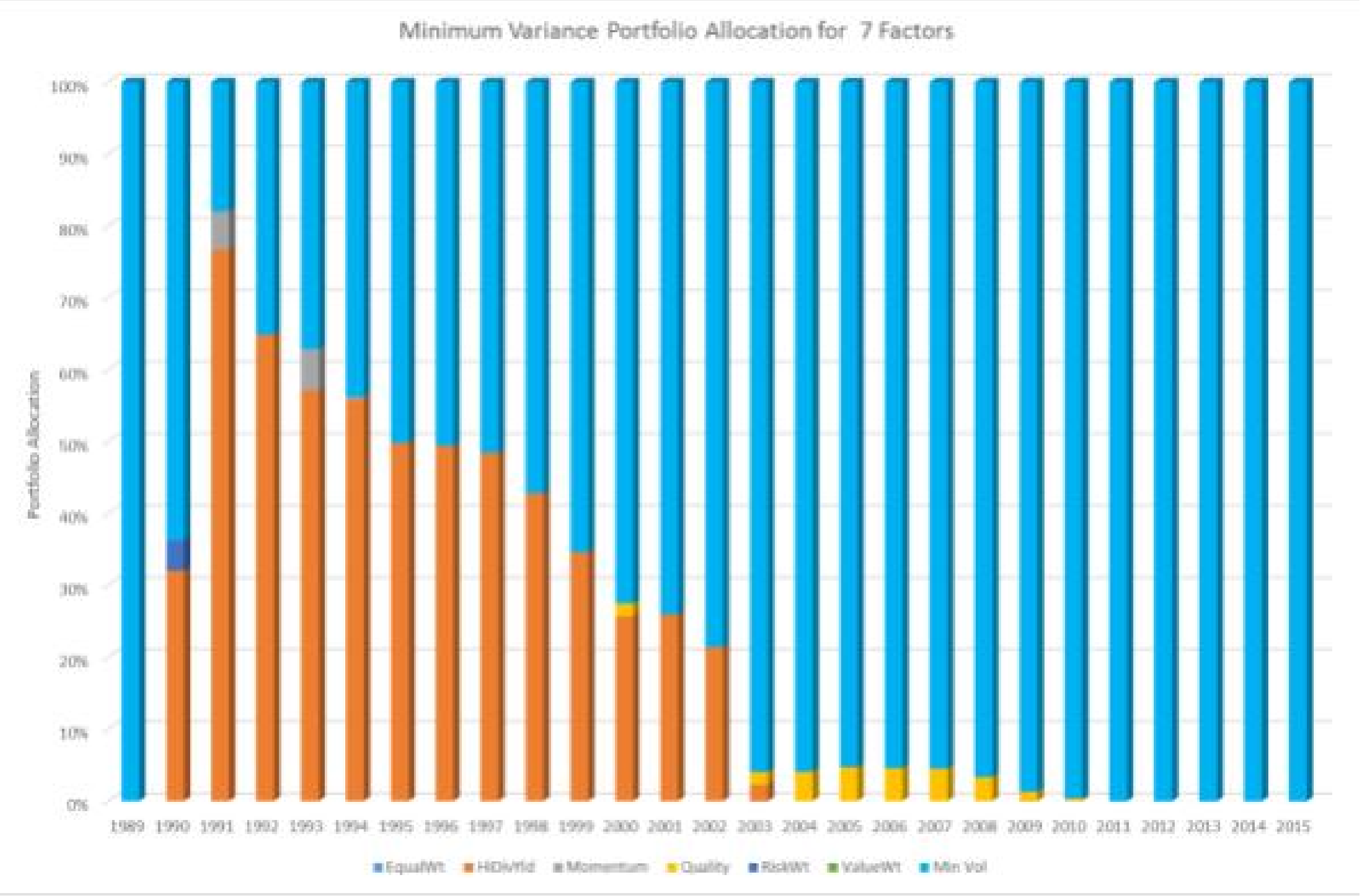
7 FACTORS: MIN VARIANCE, WHICH ONES?



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Appendix 1:
Constructing The Alternatives

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CONSTRUCTING THE SET OF ALTERNATIVES

- Equally-weighted
  - Each stock is given a weight of 1/n. This very simple and perhaps somewhat naive approach to determining weights was examined by DeMiguel, Garlappi and Uppal (2009) and found to outperform many more sophisticated methods due to the avoidance of parameter estimation errors.
- Diversity-weighted
  - This approach was first proposed by Fernholz et al (1998) Effectively it involves raising the Market-cap weight (w) of each constituent by the value p, that is w^p, where p is bounded between 1 and 0. The weight of each index constituent is then calculated by dividing its w^p weight by the sum of all w^ps of all of the constituents in the index. When p is set to 1 then the constituent weights are equal to Market-cap weights and when p is set to 0 the weights are equivalent to equal weights. We use p=0.76 which is the value used in the original paper.

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CONSTRUCTING THE SET OF ALTERNATIVES

- Inverse volatility
  - In the mid-1970s Haugen and Heins published a paper that demonstrated that low volatility stocks tended to outperform high volatility stocks, since then there has been much research on the "low-volatility anomaly". We calculate the historical return variance of each stock using five years of monthly data. We then calculate the inverse of this value, so that the stock with the lowest volatility will have the highest inverted volatility. We then simply summed these inverted variances. The weight of stock i is then calculated by dividing the inverse of its return variance by the total inverted return variance. This process therefore assigns the biggest weight to the stock with the lowest volatility, and the lowest weight to the stock with the highest return volatility.

CONSTRUCTING THE SET OF ALTERNATIVES

- Equal risk contribution
  - Maillard et al (2008) propose weighting each stock such that that the contribution of each stock to the risk of the overall portfolio is equal. We use a covariance matrix based on 5 years history (shrunk using Ledoit and Wolf) and the algorithm proposed in this paper to calculate equal risk contribution weights.
- Minimum variance
  - The minimum variance approach uses historical data in an attempt to identify the weights of the global minimum variance portfolio. Authors such as Clarke, de Silva, and Thorley (2006) have identified strong performance of minimum variance portfolios. We use the same shrunk covariance matrix as before and cap individual weights at a maximum of 5%.

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CONSTRUCTING THE SET OF ALTERNATIVES

- Maximum diversification
  - Choueifaty and Coignard (2008) introduce a measure of portfolio diversification, called the "Diversification Ratio", which is defined as the ratio of a portfolio's weighted average volatility to its overall volatility. Poorly diversified portfolios that have either concentrated weights, highly correlated holdings or even both will exhibit relatively low diversification ratios. Choueifaty and Coignard propose an optimisation process to identify the 'most diversified portfolio' which is defined as the portfolio with the highest diversification ratio. Intuitively it is apparent that if expected returns are proportional to their volatility, the maximum diversification portfolio will be the same as the maximum Sharpe ratio portfolio (this can be proven mathematically). Again we use the same shrunk covariance matrix and cap individual weights at a maximum of 5%.

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CONSTRUCTING THE SET OF ALTERNATIVES

- Risk efficient
  - Amenc, Goltz, Martellini, and Retkowsky (2010) propose a very similar methodology to maximum diversification except that they assume that the expected return on each constituent is assumed to be linearly related to the downside-deviation of its return. They also group stocks into deciles of semi-deviation and assign each stock the median of its decile. The second stage then involves finding the portfolio with the maximum expected return (proxied by the median downside deviation of each stock's decile) with the lowest portfolio return standard deviation. To prevent the optimiser from creating a portfolio with concentrated single stock exposures, they impose restrictions on the constituent weights:
    - lower limit = $1/(\lambda \times N) \times 100\%$
    - upper limit = $\lambda/N \times 100\%$

    where N represents the total number of stocks under consideration and λ is a free parameter. We set λ equal to 2 and use the same shrunk covariance matrix.

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CONSTRUCTING THE SET OF ALTERNATIVES

- Fundamental Indexing
  - Arnott et al. (2005) argue that alternative measures of the size or scale of a company may be just as appropriate a basis for determining constituent weights as the more commonly used metric of market capitalisation. We calculate four different indices that weight stocks according the 5 year historical average of total dividends, cash-flow, book value of equity and sales. We then take the average weights of these four indices to form a fundamental composite index.

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Appendix 2:
Constructing The Random Indices

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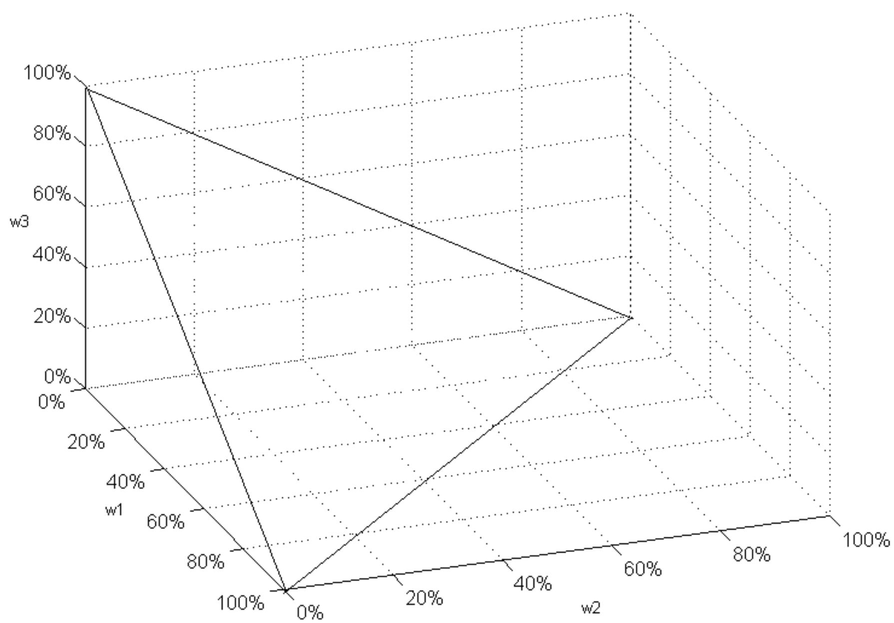
WHERE TO FIND AN INFINITE NUMBER OF MONKEYS? (NO MONKEYS WERE HARMED IN THIS EXPERIMENT)

- There are an infinite number of combinations of weights for 500 stocks that sum to 1
  - 1st step is to make this a finite universe by specifying a minimum increment Δw *of* 0.2%
  - Objective is to sample randomly and uniformly from the set of feasible weights
  - For example with 3 stocks, the set of feasible weights form a hyper-plane



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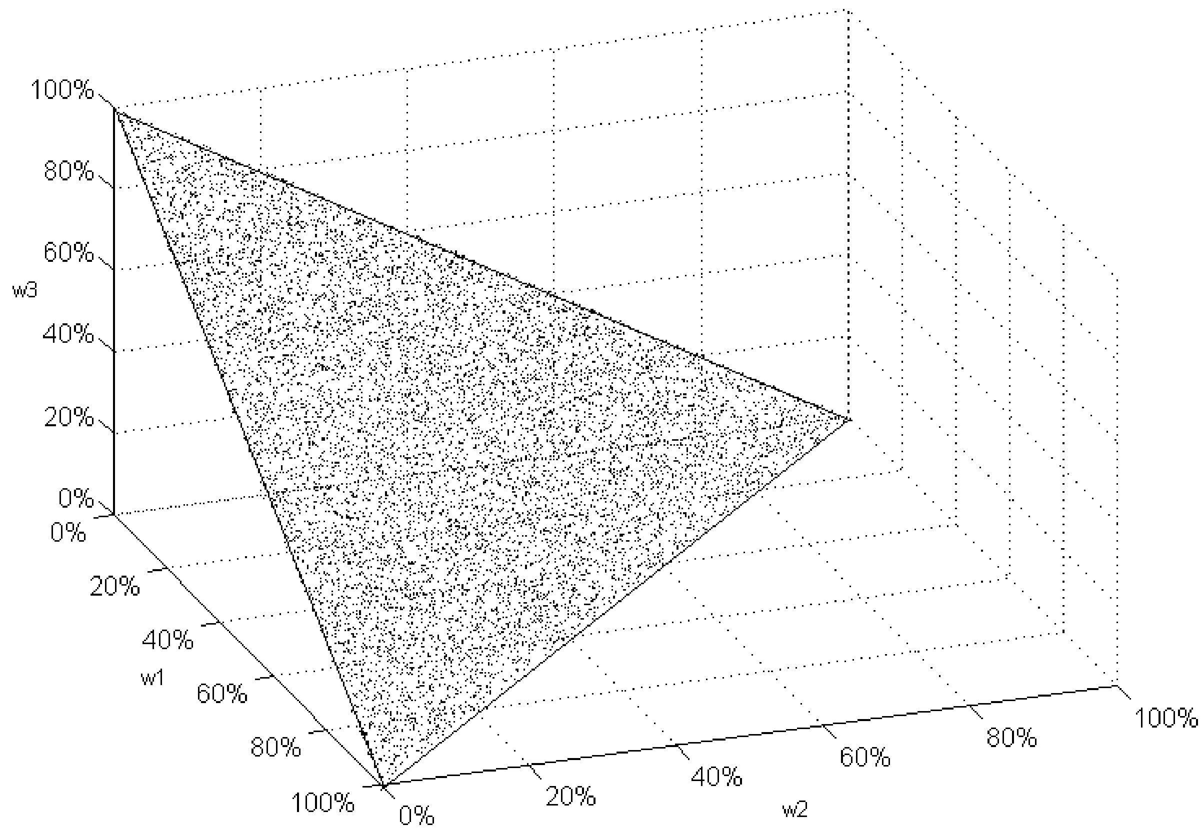
WHERE TO FIND AN INFINITE NUMBER OF MONKEYS? (NO MONKEYS WERE HARMED IN THIS EXPERIMENT)

- Use an algorithm adapted from Smith and Tromble (2004)

- Given *n* stocks, 4 steps:
  1. Sample *n-1* numbers uniformly at random from the set {1, 2, ... $(1/\Delta w)+n-1)$} *without replacement.*
  2. Sort the numbers in ascending order and append a zero to the beginning of the sequence and $(1/\Delta w + n)$ to the end of the sequence.
  3. Take the difference between successive numbers in the sample and subtract 1 from each.
  4. Multiply these numbers by Δw.

WHERE TO FIND AN INFINITE NUMBER OF MONKEYS? (NO MONKEYS WERE HARMED IN THIS EXPERIMENT)

- Scatter plot of the result of 10,000 repetitions of the above algorithm for n=3 and Δw=0.1%



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PROOF OF ROBUSTNESS

- Though the *mean* of our weights will be the same as equal weight there is no bias towards any weighting scheme:
  - Consider the example of a portfolio containing 100 stocks (n=100) where the minimum increment is set at 1% (Δw =0.01)
    - The first step involves selecting 99 random numbers from the set {1, 2, ... 199}. If we suppose that the numbers chosen are {2, 4, 6, … 198}
    - then step 2 will result in the following set of 101 numbers {0, 2, 4, 6,…..,198, 200}.
    - Step 3 produces 100 identical numbers {1, 1,….1}
    - Hence step 4 will generate an equally weighted portfolio with each stock given a weight equal to 1% or 1/n.

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PROOF OF ROBUSTNESS

- If instead the 99 random numbers chosen had been {1, 2, 3, ... 99} then the set of weights produced would be zero for the first 99 stocks and 100% in the 100th stock.

- Since choosing {2, 4, 6, ... 198} and choosing {1, 2, 3, ..., 99} are equally likely hopefully this demonstrates that the randomly generated portfolio weights are unbiased.

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CONSTRUCTING THE RANDOM INDICES

- Using the algorithm we generate 500, weights that sum to one, with a minimum increment of 0.2%.
  - Apply these weights to the universe of 500 stocks sampled at December 1968
  - Calculate the performance of the resulting index over the next twelve months.
  - Apply another set of randomly generated weights to the 500 stocks sampled in December 1969, and again calculate the performance of this randomly constructed index over the next 12 months.
  - Repeated for each year in our sample until we produce an index spanning January 1969 to December 2014.

- Repeat the whole process ten million times……….

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